GRAPHICAL APPROACH TO TEACHING COMPOUND INTEREST IN HIGH SCHOOL MATH CLASSES

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Abstract: *The topic of compound interest has its place in high school mathematics curriculum to introduce the students to one important application of exponential function. It prepares them to understand the basics of the most widely spread financial instruments like deposits and loans. Modern open source Dynamic Geometry Systems (DGS) like GeoGebra allow high school mathematics teachers to apply a dynamic graphical approach to the concept of compound interest. The elements of inquiry based learning implemented in math classes help students become independent and critical thinkers, decision makers, and discoverers. Experiments and observations with DGS serve students as a source of formulating and verifying the financial Rule of 72, the Continuous compounding concept, and their own hypotheses as well. Experience gained from the inquiry based learning activities accustoms the students to using DGS for analysis of financial situations later in life.*

Key words: *Dynamic geometry systems (DGS), Inquiry based learning, Exponential function, Compound interest, The Rule of 72, Continuous compounding*

1. Introduction

National high school mathematics curriculum considers mathematics classes as a natural environment for learning the concept of compound interest. Preliminary knowledge on exponential function, logarithms, and geometric series allows students to mathematically model financial situations including long-term deposits and loans. The formulae in their textbooks help them find the time frames of financial events, as well as the amount of money expected or needed [1, 25-29]. In such a way mathematics crucially helps family, society, and business in laying the foundations for people's financial literacy.

The effect of studying the topic of compound interest can be even greater if inquiry based learning supported by the means of dynamic geometry systems (DGS) is implemented in problem solving activities. Guided by the teacher, the students use their knowledge on functions and graphs to model and analyze financial situations and develop their creativity. Rich graphic representations which DGS offer become a source for observations, exploration, and mathematical reasoning [2]. They enable students to develop their financial knowledge and use it to invent and test strategies to financial situations without risking real money.

2. Necessity to use DGS in teaching the topic of compound interest

In regular Bulgarian mathematics textbooks the topic of compound interest is covered fully and in detail. However, it is *not* illustrated by graphs [1, 25-29], which might pose a potential difficulty to the students.

DGS provide convenient tools to fill that unintentional gap and not only make the theoretical concept easier to understand for students, but also practical financial situations simpler to model and explore. When the choice of a particular DGS is not only a question of personal preferences but of finances as well, GeoGebra [3] as open source software attracts numerous admirers. Teachers use DGS in applying a graphical approach to the topics alongside with the analytical one. Students find inquiry based learning amusing and helpful for their success in math classes, mastering the concepts studied. For this reason, GeoGebra worksheets are used to provide a combined analytical and geometric approach to the idea of compounding the interest, thus putting the students on the path of exploration.

Although not complicated, the equation of compound interest from regular math textbooks still confuses many students, especially when they are to assign specific numerical values to its parameters:

$$f(x) = P_0\left(1 + \frac{0.01r}{T}\right)^{Tx} \tag{1}$$

Here P_0 is the initial capital, deposited at *r*% annual percentage rate (APR) for *x* years and compounded *T* times per year and $f(x)$ is the future value of P_0 that is to be calculated.

Observing the impact of parameters on the behavior of $f(x)$ through its graph reveals to students the meaning of each one; then the correct values are easy to work out and substitute. Therefore, elementary, but instructive examples are to be initially offered to the students in order to support them in their efforts to comprehend the role of parameters. Here is such a one:

Example 1. The sum of $100,000 is put in a bank deposit, compounded monthly at 7.2% APR. The deposit is closed in 11 months on maturity. How can the graph in *Figure 1* be adapted to show the amount of money withdrawn?

To get the answer, the students are to fill the data into the dynamic worksheets provided with the help of sliders. What they are to guess is that since one year has 12 months, 1-month deposits can mature up to 12 times per year. Therefore, the value of parameter *T* is 12 (see the *T*-slider in *Figure 2*).

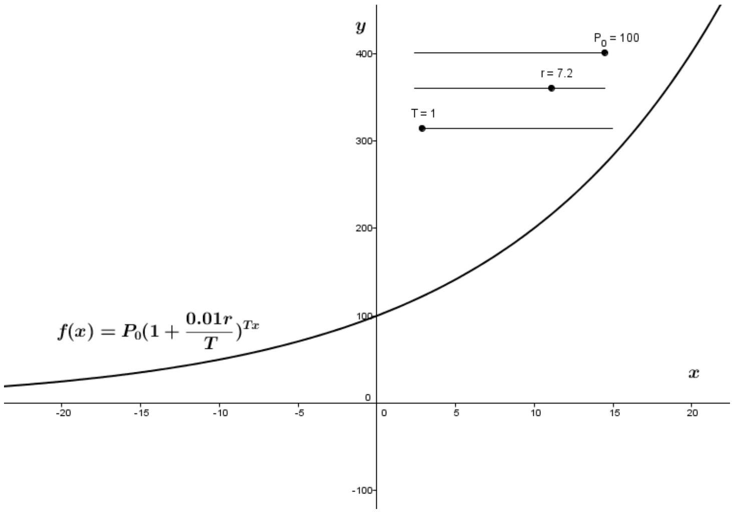


Figure 1. Exponential function used to model the time effects of compound interest: *x* – time in years, *y* – amount of money deposited in $

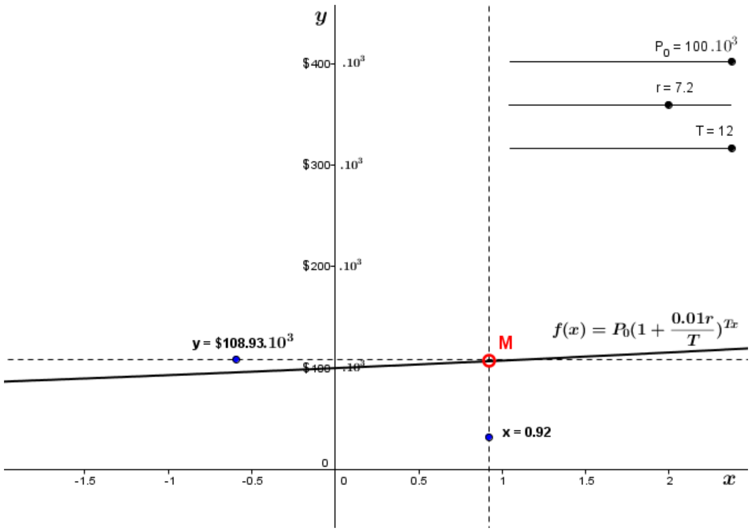


Figure 2. Exponential function as a model of a bank deposit compounded monthly for 11 months (which equal approximately a 0.92 part of a year)

The students also need a correct value for *x* to take as *x*-coordinate of a point on the graph of $f(x)$. They are to realize that the *x*-axis represents time in years and the numerical data of 11 months is to be converted to years. Therefore the *x*-value needed is $\frac{11}{12} \approx 0.92$ part of a year (*Figure 2*).

To the students whose GeoGebra worksheets have been prepared in advance by the teacher, *Example 1* offers another challenge. Those who decide to adjust the graph may not wish to change the scale on the *y*-axis although the amounts of money are 10^3 times greater. A wise move is only to change *notations of the amounts* on the *y*-axis and think of the *y*-coordinates in *Figure 1* as multiplied by 10^3. However, this would be not the case for the *x*-axis, where an appropriate enlarging of scale would provide a better view on the example's numerical data (*Figure 2*). Therefore provoking students to explore scale as a tool for quality of graphic representations develops their technical and numerical culture.

To have a dynamic worksheet better corresponding to the aims of inquiry based learning, I have imbedded two auxilliary lines which the students can slide to display the coordinates of points. The horizontal dotted line always remains parallel to the *x*-axis, while the vertical one – to the *y*-axis. Placing the vertical dotted line at position *x* = 0.92, the students find its intersection with the graph of $f(x)$. In *Figure 2*, the intersection point is denoted by *M*. Sliding the horizontal dotted line until it passes through *M* allows identifying the respective *y*-coordinate. It represents the amount of money (in thousands of dollars) withdrawn from the bank after the deposit has been closed.

The students notice that on the interval $x \in (-1.5, 1.5)$ the graph of exponential function $f(x)$ resembles a line. They explain that as a combined effect both of the slight growth of the function on the interval and of the larger scale on the *x*-axis selected for a better view. The students extend their observations and suggest that since time *x* and money *y* are nonnegative, the values of $f(x)$ are to be displayed only in the first quadrant of the coordinate system.

Compared to recent levels of APR, the data of the next example may seem a bit far-fetched. However, for educational purposes, its choice is justified:

Example 2. A bank offers 42.8% APR for 1-month deposits and 43% APR for 3-month deposits. A customer considers putting some money into either type of deposits for two years, compounding the interest to the principal. Which of the two deposits is preferable?

Without doing calculations by hand, via GeoGebra spreadsheets the students analyze the two types of deposits. It appears that although the 3-month deposits look more attractive, the effect of compounding on a long-term basis leads to a lower

APR (*Figure 3*). This counter-intuitive result shows the students the power of using math and IT to check financial operations before performing them.

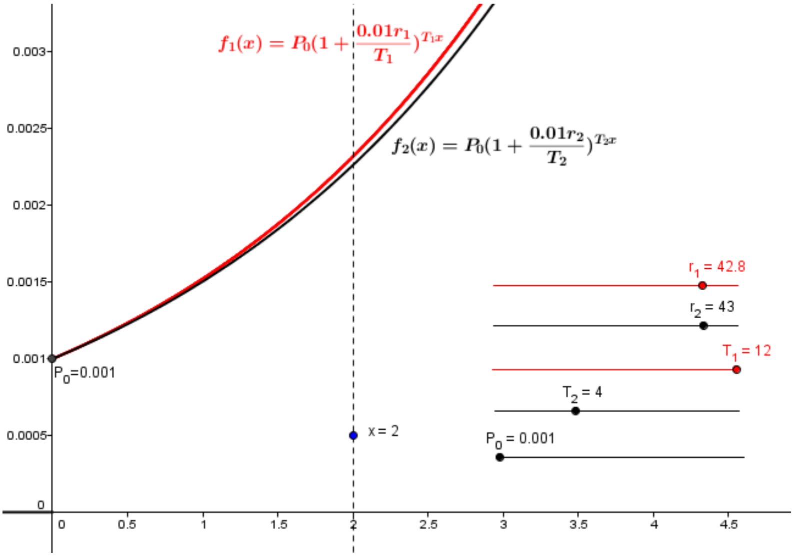


Figure 3. In long term, a higher compound interest may lead to a lower income. The graphs show that in 2 years of compounding, a 3-month deposit at 43% APR is less profitable for the customer than a 1-month deposit at 42.8% APR.

By revealing intriguing phenomena of compound interest, experiments of the kind boost students' enthusiasm and interest in finance. They also raise their responsibility to their pocket money and can even make them consider a prospective profession in finances [4-5].

3. Inquiry based learning activities

Testing the role of each parameter in equations like (1) is a regular strategy in inquiry based learning. For the teachers, it requires preliminary DGS worksheet preparing and experiments planning. For the students, it is an activity of gathering observations, formulating hypotheses, and reasoning on them. The more grounded in real finance it is, the more excited the students are by the concept of compound interest and more willing to apply it in the future.

3.1. Exploring the role of parameter r

Changing the value of parameter *r* accompanied by simultaneous tracing of the graph of $f(x)$ offers young people interesting material for interpretation (*Figure 4*).

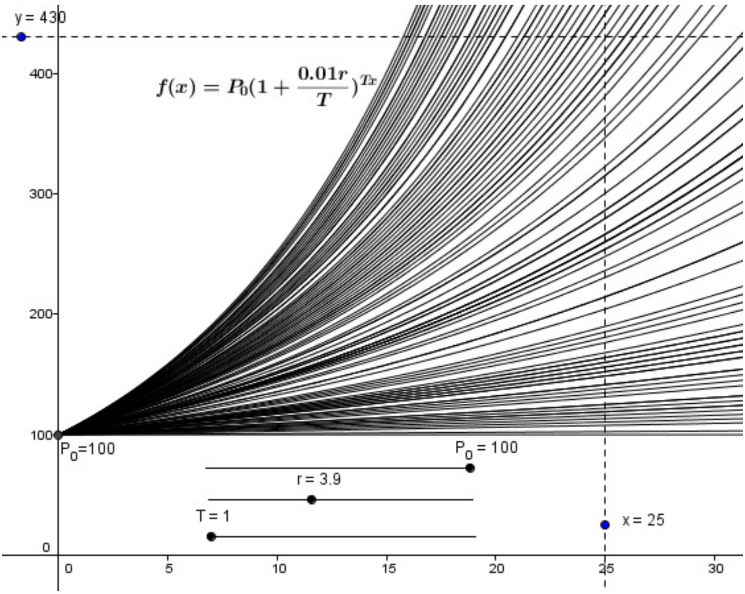


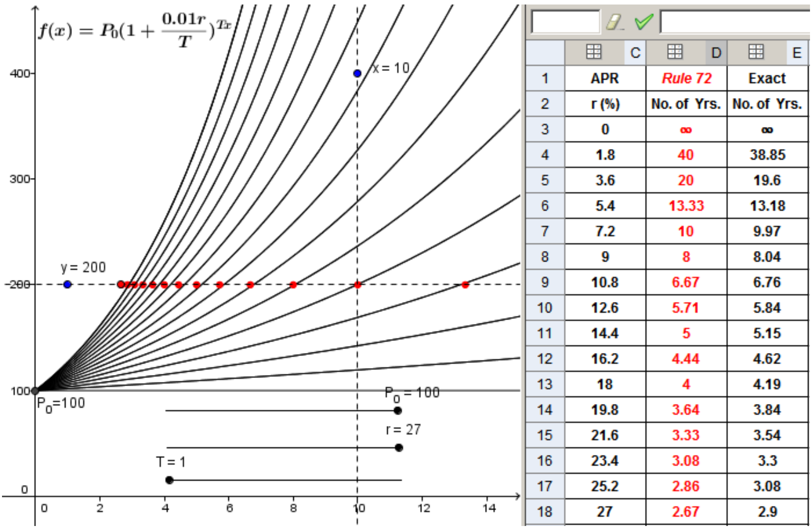
Figure 4. Changing parameter *r* from 0 to 10: the value *r* = 0 models the modern trend of lowering the deposits APR by most European banks

Looking at the exponential growth of $f(x)$, the students speculate that *the bigger the APR, the steeper the graph,* therefore, *the faster the deposited money grows*.

The limit case *r = 0* also attracts their attention because it is close to reality: many banks have recently lowered their APRs. As interest is just a price paid to the depositors for using their money, this trend should not surprise. The price of any commodity depends on the demand and supply at the free market, therefore the price of money also obeys its law *the bigger the supply, the smaller the demand, the lower the price*.

The students speculate that for the bank customers with deposits, low APRs bring almost no profit. This is not the case if money is used for long-term investments at the stock market.

Experimenting with DGS worksheets helps students discover the empirical *Rule of 72*, used for manual calculation of the number of years *x*, needed to double the money, put in a 1-year deposit at *r*% compound interest. According to the rule, *x* is approximately equal to the ratio of number 72 and number *r*. Through GeoGebra spreadsheets (*Figure 5,* right) the result can be easily compared with the exact number of years $x = \dfrac{\ln 2}{\ln(1+0.01r)}$ for which money in a 1-year deposit doubles.



	C	D	E
1	APR	*Rule 72*	Exact
2	r (%)	No. of Yrs.	No. of Yrs.
3	0	∞	∞
4	1.8	40	38.85
5	3.6	20	19.6
6	5.4	13.33	13.18
7	7.2	10	9.97
8	9	8	8.04
9	10.8	6.67	6.76
10	12.6	5.71	5.84
11	14.4	5	5.15
12	16.2	4.44	4.62
13	18	4	4.19
14	19.8	3.64	3.84
15	21.6	3.33	3.54
16	23.4	3.08	3.3
17	25.2	2.86	3.08
18	27	2.67	2.9

Figure 5. The *Rule of 72* illustrated: finding the approximate number of years needed to double money P_0 in a 1-year deposit at r% compound interest

3.2. Exploring the role of parameter T

Another group of experiments the students perform is based on changing the value of parameter *T* through the slider. The graphs of functions $f(x)$ obtained through DGS for different values of *T* are shown in *Figure 6*.

Enlarging the number of compounding periods introduces the concept of continuous compounding to the students: from one per year (*T* = 1) to one per hour every day of the year (*T* = 8,760) to every moment of the time as an abstract idea. Through the limit of function, it leads to transition from $f(x) = P_0\left(1+\dfrac{0.01r}{T}\right)^{Tx}$ to a new function $q(x) = \lim_{T\to\infty} f(x) = P_0 e^{0.01rx}$ describing continuous compounding. Its graph is shown in *Figure 6* and some of its values – in the bottom row of *Table 1*.

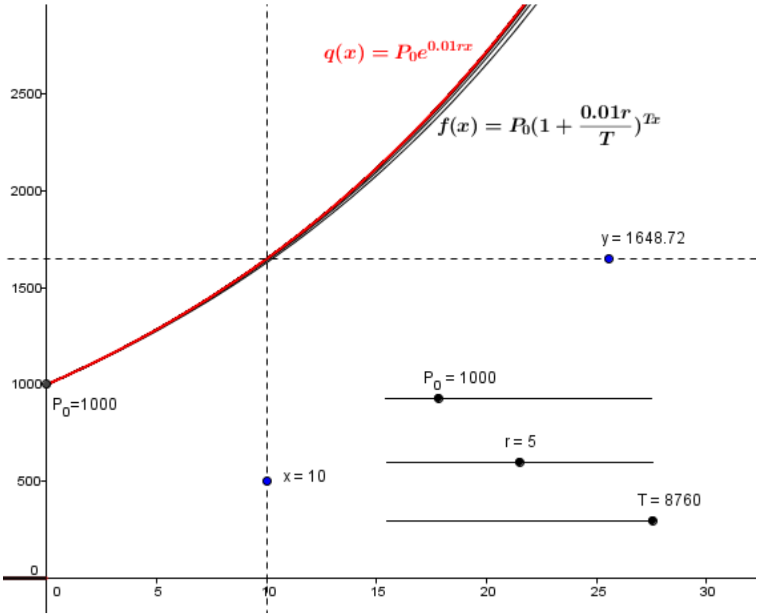


Figure 6. Comparison between discrete and continuous compounding by the graphs of functions $f(x)$ and $q(x)$ respectively

Table 1. Final amount of \$1,000 deposited at 5% compound interest

Compounding	Value of T	in 1 year	in 10 years	in 20 years
Every year	1	\$1,050.00	\$1,628.89	\$2,653.30
Every month	12	\$1,051.16	\$1,647.01	\$2,712.64
Every hour	8,760	\$1,051.27	\$1,648.72	\$2,718.27
Continuous	∞	\$1,051.27	\$1,648.72	\$2,718.28

4. Conclusion

The modern financial world is full of mathematical ideas, models, and algorithms. Although not complicated, the concept of compound interest is among the most important of them. Learned during the school years, it lays the basics of students' mathematical and financial literacy. Thus mathematics provides practical knowledge aimed at meeting students' present and future financial needs, from management of their pocket money and credit cards through savings accounts and deposits to stock market investments.

Studying compound interest by means of DGS makes the concept vivid, easy to understand, remember, and use by the students. Having solved financial problems with DGS can be real help in negotiating deals; approaching risky financial situations with the power of mathematical knowledge and IT essentially improves the decision making and the choice of a strategy. Therefore, financial literacy based on studying compound interest in schools should be considered an essential key competence in mathematics education both of students and teachers in the frame of the European Project *KeyCoMath* [6].

Acknowledgements

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Note

Dynamic worksheets shown in Figures 1 through 6 are made for educational purposes with the help of GeoGebra DGS [3].

References

1. Lozanov, C., Vitanov, T. & Nedevski, P. *Mathematics for Eleventh Grade: Advanced Course*. Sofia, Bulgaria: Anubis. 2001. (in Bulgarian)
2. *Principles and standards for school mathematics*. Reston, VA: NCTM, 2000.
3. *GeoGebra* dynamic mathematics software https://www.geogebra.org (accessed 2015-04)
4. Gortcheva, I. Financial education through mathematics and IT curricula: pocket money management. In Totkov, G., & Koychev, I. (Eds.), *Proceedings of the 6th National Conference “Education and Research in Information Society”*. Plovdiv, Bulgaria: Association for Development of Information Society, 2013, 49-57.
   http://sci-gems.math.bas.bg:8080/jspui/bitstream/10525/1999/1/EIS2013-book-p04.pdf
   (accessed 2015-04)
5. Gortcheva, I. Developing financial expertise through mathematics and IT school curricula. In Totkov, G., & Koychev, I. (Eds.), *Proceedings of the 7th National Conference “Education and Research in Information Society”*. Plovdiv, Bulgaria: Association for Development of Information Society, 2014, 48-57. (in Bulgarian)
   http://sci-gems.math.bas.bg:8080/jspui/bitstream/10525/2394/3/ERIS2014-book-p05.pdf
   (accessed 2015-04)
6. *Developing Key Competences by Mathematics Education*: European project KeyCoMath. http://www.keycomath.eu/ (accessed 2015-04)

ГРАФИЧЕН ПОДХОД ПРИ ПРЕПОДАВАНЕ НА СЛОЖНА ЛИХВА В УЧИЛИЩЕ

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Резюме: *Темата за сложна лихва, изучавана в часовете по математика в горния курс, запознава учениците с това важно приложение на показателната функция и ги подготвя да разберат основни финансови инструменти като депозити и заеми. Съвременните системи за динамична геометрия, каквато е GeoGebra например, позволяват на учителите по математика да реализират графичен подход в уроците за сложна лихва. Елементите на изследователския подход допринасят съществено за изграждане на собствено критично мислене у учениците. Експериментите и наблюденията, осъществени с динамичен софтуер, им позволяват да формулират и проверяват свои и класически финансови идеи, сред които са непрекъснатото олихвяване, Правилото 72 и др. Опитът, придобит от учениците в резултат на изследователския подход и работата им със системи за динамичен софтуер е приложим и след завършване на училище за анализиране на реални финансови ситуации в живота.*